AN EFFICIENT TRAINING METHOD FOR QUALIZATION OF DISCRETE MULTITONE TRANSCEIVERS

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ABSTRACT

In Discrete Multitone (DMT) transceivers, a cyclic prefix is inserted between transmitted symbols such that the linear convolution of the data and the channel impulse response becomes a circular one corresponding to term-by-term product in the frequency domain. If the cyclic prefix length is longer than that of the channel impulse response it avoids the intersymbol interference (ISI). To reduce the inefficiency due to the use of a long cyclic prefix, the use of a time domain equalize. (TEQ) to shorten the effective channel impulse response has been the most popular equalization approach. In this paper, we pose the TEQ problem completely in the frequency domain thus increasing the efficiency in computation and implementation. We derive anew TEQ training algorithm by minimizing a squared cost function defined in the frequency domain. Simulation results show that this new algorithm outperforms Amati's method by resulting in smaller residual intersymbol interference at a reduced computational cost.

I. INTRODUCTION

Multi-carrier modulation (MCM) has been proposed for parallel communication in the late 1950s [1] based on the concept of creating multiple orthogonal subchannels over which several data streams can be sent without intersymbol interference (ISI). This modulation scheme provides flexibility for adapting to different channel environments by adjusting the energy and constellation size of each carrier. One implementation of MCM is the Discrete Multi-tone(DMT) system which uses the discrete Fourier transform(DFT) for modulation [2], [3]. DMT has recently been chosen as the industry modulation standard for Asymmetrical Digital Subscribe. Loop (ADSL) modems, which offer powerful and flexible transmission capability enabling delivery of a variety of multimedia services over the existing telephone networks.

The block diagram of the DMT system is shown in Fig. 1. The input data stream is coded by forward-error-correcting (FEC) and/or trellis coding schemes. The outputs of the encoder are grouped into QAM subsymbols. A complex-to-reat inverse fast Fourier transform (IFFT) used for modulation is performed to convert these QAM subsymbols to real ones. Then the last Y samples of each real-valued data vector are copied and prefixed to the data vector. At the receiver, the channel outputs are passed through the time domain equalizer (TEQ). After removing the samples corresponding to the cyclic prefix, the outputs of the TEQ are processed by the fast fourier transform (FFT) which acts as the demodulation operation. Then the transmitted data can be recovered by using the adaptive frequency domain equalize. (FEQ) followed by a decoder.

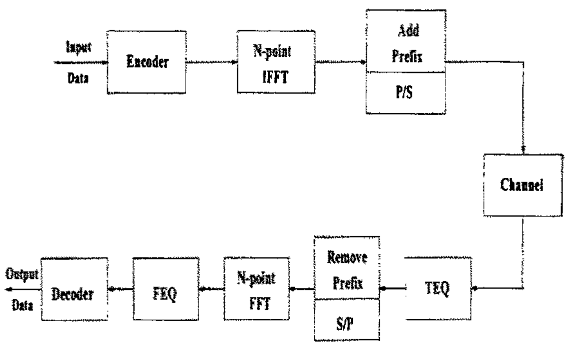

Figure 1: Block diagram of the DMT system

In the DMT system, the cyclic prefix provides a guard time between blocks. If the ISI of the channel does not extend beyond the cyclic prefix length Y, DMT symbols can be transmitted free of ISI. Using large Y values to compensate for the length of the channel response, however, decreases the efficiency (introduces an overhead of Y/(N + Y)) and increases latency. A time domain equalizer to shorten the effective channel impulse response has been the most popula. equalization approach for DMT [4], [5], [6], [7]. This approach uses the fact that the channel transfer function in DMT system can be approximated by an autoregressive moving average (ARMA) model:

$$H(z^{-1}) = \frac{B(z^{-1})}{A(z^{-1})} = \frac{z^{-d}\sum_{i=0}^{L-1} b_i z^{-i}}{1+\sum_{i=1}^{M-1} a_i z^{-i}} \qquad (1)$$

A TEQ whose transfer function is equal to $A(z^{-1})$ can be introduced at the receiver side, such that the cascade of channel and TEQ produces a sufficiently short impulse response $B(z^{-1})$. In Eqn. (1), L is the length of the desired shortened impulse response where L should be less than or equal to Y+1, M is the length of the TEQ filter, and d is the delay of the equalized channel response. Usually, the channel impulse response can not be perfectly shortened by the TEQ such that some residual energy of the shortened impulse response will lie outside the Y+1 consecutive taps with the highest total eaergy. We can use the shortening signal to noise ratio (SSNR) [6] to measure the convergence of the TEQ. This SSNR is defined as the ratio of the energy in the largest consecutive Y+1 taps to the energy in the remaining taps of the equalized response. The TEQ methods given in [5] and [6] use cost functions that minimize the residual error in the time domain. These methods require the computation of the auto correlation and cross correlation coefficients of the transmitted and the received signals. Because of severe frequency selective channel distortion, the input to the equalizer at the receiver will be highly correlated causing the conventional time domain least mean squares (LMS) type algorithms to suffer from slow convergence. The group at Amati Communication Corp. (Chow *et al.* [4]) proposed an algorithm which minimizes the mean squared error of the equalized response by using frequency domain LMS for the adaptation and windowing in the time domain, However, the error signal used for the adaptation is in the frequency domain. The convergence of Amati's algorithm is slow [5], which is also confirmed by our simulation results. In this paper, we pose the TEQ problem completely in the frequency domain. The advantage is that the DFT can provide good orthogonalization to the highly correlated received signal due to the severe frequency selection of the channel in the DMT system. the introduce the weighted frequency domain least squares (WFD-LS) method and derive the corresponding algorithm by minimizing a squared cost function defined in the frequency domain. We then present simulation results and show the superior performance of the WFD-LS algorithm compared to Amati's method in terms of computational complexity and residual intersymbol interference.

2. WEIGHTED FREQUENCY-DOMAIN LEAST SQUARES ALGORITHM

During the initial training phase in a DMT transceiver, a pseudo-random sequence is transmitted repeatedly over the channel to form a periodic signal. Suppose $X(e^{-jw})$ and $Y(e^{-jw})$ are the DFTs of the training and the received signals, then the channel frequency response can be estimated as:

$$H(e^{-j\omega}) = \frac{Y(e^{-j\omega})}{X(e^{-j\omega})} \equiv \frac{B(e^{-j\omega})}{A(e^{-j\omega})} \qquad (2)$$

We would like to model the channel as the ratio of $B(e^{-jw})$ and $A(e^{-jw})$, where $A(e^{-jw})$ can be used as the TEQ. If there is no error in the modeling, we achieve:

$$B(e^{-j\omega})X(e^{-j\omega}) = A(e^{-j\omega})Y(e^{-j\omega}) \qquad (3)$$

Instead of using the difference between $B(e^{-jw})X(e^{-jw})$ and $A(e^{-jw})Y(e^{-jw})$ as the cost function which is used in Amati's algorithm [4], we introduce the following squared cost function:

$$E(\theta) = \sum_{k=0}^{N/2} W(k)|H(e^{-j\omega_k}) - \frac{B(e^{-j\omega_k})}{A(e^{-j\omega_k})}|^2 \qquad (4)$$

where $\Theta^T = (a_1,\ldots,a_{M-1},\ b_0,\ldots,b_{L-1})$ is the parameter vector of all the parameters of the ARMA model to be estimated, N is the FPT length, k is the index of the subchannel frequencies, and $W(k)$ is a positive weighting function which can be used to incorporate frequency domain constraints.
However, the cost function $E(\Theta)$ is nonlinear in the parameter vector Θ, which increases the complexity of the optimization procedure to compute the parameters. We use its linear least squares estimator E_{ls} whose unweighted version is used in [8] for a continuous-time system:

$$E_{ls}(\theta) = \sum_{k=0}^{N/2} W(k)|H(e^{-j\omega_k})A(e^{-j\omega_k}) - B(e^{-j\omega_k})|^2 \quad (5)$$

If we differentiate $E_{ls}(\Theta)$ with respect to each unknown coefficient a_i, and b_i, and set the result to zero, we obtain:

$$\Phi\eta = \beta \qquad (6)$$

where

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} \tag{7}$$

$$\Phi_{11} = \begin{bmatrix} \mu_0 & \mu_1 & \cdots & \mu_{M-2} \\ \mu_1 & \mu_0 & \cdots & \mu_{M-3} \\ \vdots & & & \vdots \\ \mu_{M-2} & \mu_{M-3} & \cdots & \mu_0 \end{bmatrix}$$

$$\Phi_{12} = \begin{bmatrix} -p_{d-1} & -p_d & \cdots & -p_{d+L-2} \\ -p_{d-2} & -p_{d-1} & \cdots & -p_{d+L-3} \\ \vdots & & & \vdots \\ -p_{d-M+1} & -p_{d-M+2} & \cdots & -p_{d+L-M} \end{bmatrix}$$

$$\Phi_{21} = \begin{bmatrix} p_{d-1} & p_{d-2} & \cdots & p_{d-M+1} \\ p_d & p_{d-1} & \cdots & p_{d-M+2} \\ \vdots & & & \vdots \\ p_{d+L-2} & p_{d+L-3} & \cdots & p_{d+L-M} \end{bmatrix}$$

$$\Phi_{22} = \begin{bmatrix} -\lambda_0 & -\lambda_1 & \cdots & -\lambda_{L-1} \\ -\lambda_1 & -\lambda_0 & \cdots & -\lambda_{L-2} \\ \vdots & & & \vdots \\ -\lambda_{L-1} & -\lambda_{L-2} & \cdots & -\lambda_0 \end{bmatrix}$$

$$\eta = \begin{bmatrix} a_1 & a_2 & \ldots & a_{M-1} & b_0 & b_1 & \ldots & b_{L-1} \end{bmatrix}^T \tag{8}$$

$$\begin{aligned} \beta = \quad & [-\mu_1 - \mu_2 \ldots - \mu_{M-1} \\ & -p_d - p_{d+1} \ldots - p_{d+L-1}]^T \end{aligned} \tag{9}$$

and

$$\mu_i = \sum_{k=0}^{N/2} |H(e^{-j\omega_k})|^2 \cos(i\omega_k) W(k), \tag{10}$$

$$p_i = \sum_{k=0}^{N/2} [R_k \cos(i\omega_k) - I_k \sin(i\omega_k)] W(k), \tag{11}$$

$$\lambda_i = \sum_{k=0}^{N/2} \cos(i\omega_k) W(k), \tag{12}$$

$$R_k = Re\{H(e^{-j\omega_k})\}, \quad I_k = Im\{H(e^{-j\omega_k})\}, \tag{13}$$

Hence solution of the set of linear equations shown in (6) yields the optimal parameters a_i (i=1,...,M-1) and b_i (i=0,...,L-1) such that the linearized frequency domain approximation error square (5) is minimized. The resulting algorithm is called the weighted frequency domain least squares (WFD-LS) algorithm. In practice, matrix inversion is seldom used to solve linear equations of the type (6) due to low numerical accuracy and long execution time. In this paper, we use Gaussian elimination to solve (6) [9]. Assume that L = M - 1 in (7) and the computational requirement of division is equivalent to that of multiplication, then the computational complexity for WFD-LS algorithm in terms of real multiplications carl be estimated as follows [9] :

1. $5(L+1) \times (\frac{N}{2}+1)$ multiplications to compute μ_is(i=0,...,M-1); $10L \times (\frac{N}{2}+1)$ multiplications to calculate p_is (i=d-M+1,..., d+L-1);$2L \times (\frac{N}{2}+1)$ multiplications to compute λ_is (i=0,...,L-1).
2. About $\frac{(2L)^3}{3}$ multiplications to convert the matrix ϕ in (7) into an upper triallgular one.
3. About $2L^2$ multiplications to obtain the unknown coefficients in (6) by solving the linear equations with upper triangular matrix.

Then, altogether we need $(17L+5) \times (\frac{N}{2} + 1) + \frac{8}{3} L^3 + 2L^2$ multiplications for implementing the WFD-LS algorithm. For example, for the upstream data transmission in the DMT system, when L equals to 5 and N is 64, the number of multiplications is 3,353.

3. SIMULATION RESULTS

In this section, we compare the performance of the WFD-LS algorithm with that of Amati's algorithm. We consider the transmission of the training sequence specified in the ADSL standard [10] for upstream data through the channels shown in Fig. 2. The impulse response shown in Fig. 2 is a truncated ARMA channel with denominator order 5 and numerator order 4, which is produced using the filter function in Matlab. The actual coefficients of the ARM, model is specified in the second column of Table 1. For this case, the number of used sub channels is considered to be 32, and the length of the cylic prefix, Y, 4. In this simulation, we choose L = 5 and M = 6 for both the WFD-LS and Amati's algorithm. Because we can mitigate the noise contribution during the start-up training phase by averaging multiple received signal blocks, we don't consider additive noise effect in the channel in our simulations for simplicity.

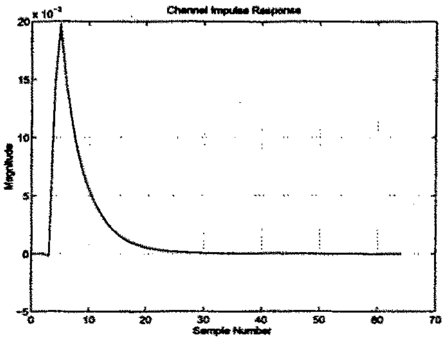

Figure 2: Channel impulse response

Table 1: ARMA(5,4) model channel identification results using WFD-LS algorithm

θ	Actual values	Estimated values
$d(delay)$	2	2
b_0	-1.888115×10^{-4}	-1.888423×10^{-4}
b_1	1.478744×10^{-2}	1.478740×10^{-2}
b_2	-4.749041×10^{-3}	-4.747862×10^{-3}
b_3	-4.461853×10^{-3}	-4.461838×10^{-3}
b_4	3.254808×10^{-3}	3.254305×10^{-3}
a_1	-1.682530×10^{0}	-1.682449×10^{0}
a_2	9.765285×10^{-1}	9.764197×10^{-1}
a_3	-1.933608×10^{-1}	-1.933284×10^{-1}
a_4	-1.521425×10^{-2}	-1.521182×10^{-2}
a_5	-1.162625×10^{-3}	-1.162702×10^{-3}

First, we use the WFD-LS algorithm with weighting function $W(k)$ equal to $1/|H(e^{-jwk})|^2$ to estimate the coefficients of this channel. The estimated coefficients of this ARMA model channel are shown in the third column of Table 1. We can see that these estimated coefficients are very close to the actual ones. The relative error values for all the estimated coefficients of the ARMA channel are less than 0.025%. We also use Amati's algorithm with the same modeling orders as those of WFD-LS for estimating the coefficients of the ARMA channel. The resultant values after 200 iterations for shortening the channel response are completely different from the actual ones as also observed in Fig.3. Fig. 3 shows the combined channel and TEQ impulse responses obtained by WFD-LS and Amati's algorithm. The combined channel and the TEQ impulse response is the linear convolution of the channel response and the calculated TEQ values. Note that when we implement Amati's method, we use the initial condition a = (1.0,0.0,...,0.0) and window the b and a vectors starting from the third and the first taps respectively. The SSNR for the equalized channel is 118.22 dB by using WFD-LS algorithm, while the SSNR obtained by Amati's algorithm is 47.74 dB

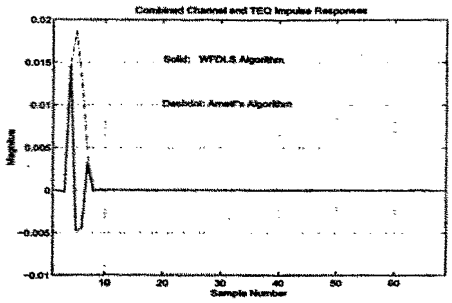

Figure 3: Combined channel and TEQ impulse responses

From the description of the WFD-LS algorithm in Section 2, WFD-LS method only needs to solve a set of linear equations for one iteration. When L equals to 5 and , N is 64, the number of multiplications is 3,353. We can also estimate the computational requirement of Amati's algorithm in terms of the number of real multiplications and divisions. Assume that the FFT is implemented by using decimation-in-time version of Radix-2 FFT algorithm[11], the computational complexity of division is same as that of multiplication, and N is the number of points of FFT and IFFT, then each iteration of Amati's algorithm needs $8N \log_2 N + 11N$ multiplications. For the upstream data transmission in the DMT system, where N equals to 64, the total number of multiplications for each iteration is 3,776, which is more than all the computational requirement of the WFD-LS algorithm in terms of multiplications and divisions. In this simulation, 200 iterations are needed for shortening the channel response shown in fig. 2 and the resulting combined response is shown in Fig. 3.

In the second set of simulations, we compare the Amati's algorithm and WFD-LS with weighting function W(k) equal to $1/|H(e^{-jwk})|^2$ for the downstream channel. The corresponding channel response and squared transfer function are shown in Figs. 4 and 5. We first transmit the training sequence specified in the ADSL standard [10] for down stream data through the channel shown in Fig. 4, then get the estimate of the channel transfer function. The additive noise effect is also ignored in this part for simplicity. We choose L = 33 and M = 12 for both WFD-LS and Amati's algorithm. The delay of the shortened channel response for Amati's algorithm is automatically determined during the iterations, After 500 iterations, the SSNR for the resultant shortened response obtained by Amati's algorithm is 40.1 dB. In WFD-LS algorithm, we note that unsuitable choice of the delay d could cause a performance degradation. So we search for the optimal delay of the shortened response before the first peak of the channel response shown in Fig. 4. We can see from Fig. 5 that some frequency subchannels of the channel transfer function have high attenuations. To avoid numerical difficulties in the implementation of the WFD-LS algorithm due to the frequency bins with very small energy, we ignore them during the computations. The SSNR obtained by WFD-LS algorithm for the channel shown in Figs. 4 is 62.5 dB, which is much higher that obtained by Amati's algorithm..

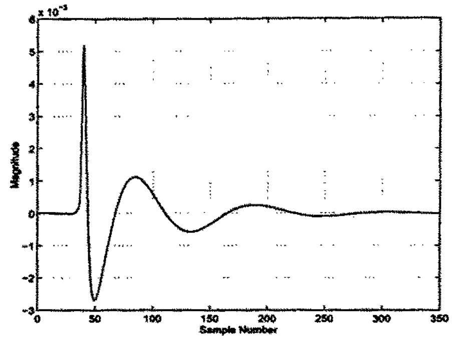

Figure 4: Channel impulse response

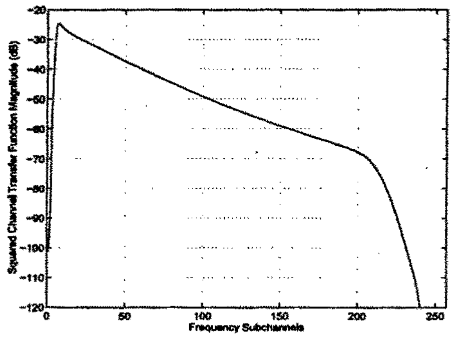

Figure 5: Channel transfer function

4. CONCLUSIONS

In this paper, we propose a weighted frequency domain least squares method to estimate the coefficients of the time domain equalizer of the DMT system such that the channel impulse response is shortened. It is shown by simulations that WFD-LS can estimate the coefficients of ARMA modeled channels much more accurately compared to Amati's method given that the correct orders and the delay of ARMA models are obtained. Furthermore, with the same length, the TEQ obtained by WFD-LS will produce much smaller inter symbol interference than the corresponding one produced by Amati's algorithm.

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